**Magnetic Susceptibility**

We’ll take a look at the dynamic magnetic susceptibility. Basically same considerations as in the time-independent case. Only now, the system won’t be able, presumably, to settle into thermodynamic equilibrium. We’ll have to use non-equilibrium statistical mechanics to calculate these response functions.

**Magnetic Susceptibility Definitions**

So we have the current induced by the magnetization,



We can take the Fourier transform of both sides,



Normally, we’d be definining,



And we still do. This definition doesn’t really presume interacting dipoles. For instance, the magnetization of a single dipole would still have to saturate, which phenomenologically wouldn’t happen if M were simply proportional to B. Then it could grow without bound as B does. Taking the Fourier transform. We have:



Then using,



And plugging this equation into the 1st we have:



and so,



**Putting χm in terms of GF**

Would like to work out a GF expression for the magnetic susceptibility. Our Hamiltonian for the interacting electron model in a magnetic field and jelly would be, in the position basis (last two V terms comprise e-e, e-jelly + jelly-jelly):



where **S**σσ´ = (1/2)[σx**i** + σy**j** + σz**k**] and σσ´ are indices and σx,y,z are the Pauli-spin matrices (see the 2nd quantization file for the spin part). But note it is the *total* interstitial B and A which enter into the Hamiltonian, not the external B and A. Reprising our work in the Conduction file to simplify the first part, we find:



where,



Then recalling,



we can translate things to the momentum basis,



We’d probably neglect the A2 term, at least if we’re just interested in susceptibilities (cause we only care about going to first order). So then,



Now we’re going to specialize to study just the paramagnetic effect, so we’ll drop the diamagnetic term (perhaps we’re implicitly presuming the presence of disorder which would severely dampen the diamagnetic response of free electrons?).



Now the magnetic field, **B** = **B**f + **B**b, is the total field coming from both the external source (f) and the electrons themselves (b). But if we’re just considering the electrons as the system, then really, it seems, we should put H in terms of the only independent d.o.f., which is **B**f. That means we need to put Bb in terms of Bf and/or the electronic d.o.f. And to do this right (see Stat Mech/Quantum MFT Paramagnet) I think this would require putting the dipole-dipole interaction between spins in the Hamiltonian H as well, and maybe doing some sort of Mean Field Approximation on H. But (see Stat Mech/Quantum MFT Paramagnet again) I think we can avoid all that by instead encompassing within our H the free magnetic field. Then our Hamiltonian for electrons and free field would be:



And so then **B**(q,t) is a free variable itself. And a derivative of H w/r to **B** would give us **H** = **B**/4π – **m** (see EM/Insulator Energy, and **m** is magnetization density, also written as **M** often times). Working out this derivative, we then see that **m** is just the coefficient of the linear **B** term, i.e. the spin term. And in position space this is:



FWIW, a Fourier transform gives:



I’ll actually write this in terms of the spin number density, which was introduced in the Collective Oscillations file (spin)



And in terms of this, the perturbation may be written (implicit summation over σσ´)



We’d like to calculate the spin number density’s expectation as a function of time. To do this we must take the trace of it with the nonequilibrium distribution function. We’ll basically rederive Kubo’s formula from the Stat Mech folder.



(where Ueq is the time-development operator according to Heq which is the H w/o VB, and by VB(Heq)(t) = Ueq†VB(t)Ueq I mean the operator in the interaction picture). Continuing,



The exponential infinitesimal convergence factor in f1, designed to prevent recurrences, limits the applicability of our results to the thermodynamic limit, because only here would energy spacings be infinitesimally small. Now we’ll plug in our expression for VB(Heq)(t)…in position space, cause…



Of course we can recognize a GF here – the (magnetization) density, (magnetization) density correlation function. And so we can say, isolating the steady state response by taking t0 → ∞:



where,



which was introduced in the Collective Oscillations (spin) file. I think that it’s most typical to simply look at spin correlations in the same direction – like zz. If we look in the Collective Excitations folder, we’ll see the Feynman diagram rules for this guy. Anyway, going back to:



and taking the Fourier transform, we come to:



Multiplying both sides by gμB, we have:



Now from the EM folder/Insulators or whatever, we’ll recall [faux Gaussian units],



So,



The division by the tensor must be interpreted as multiplying (from the left), by the matrix inverse of that tensor. And so we have, filling this into the above m = -Π(gμB)2B equation:



And that makes the susceptibility equal to:



Again, have to interpret the division as a matrix inverse. Let’s suppose the **Π** tensor is isotropic. Then it would be diagonal with all components equal. We’ll single out the ΠsR(zz)(q,ω) component, and say:



And so we’d have:



**Appendix: Consequences of Spin Rotation Symmetry**

Can refer to Quantum Mechanics/Identical Particles/2nd Quantization in Position space file for the effects of symmetry operators on creation/annihilation operators. We’ll take a look at the spin-rotation operator. Doing a 180o rotation about the y-axis, as the following effect,



Applying this to our GF, and noting that we can slide D, D† past the Hamiltonian b/c Heq (which doesn’t include the perturbative **S**·**B** term) is invariant w/r to rotations.



Now consider π rotations about the x-axis. We found,



Applying to our GF,



Comparing the two, we see we must have:



So could say whatever sign the product of the incoming pair of spins has, the outgoing product must have the same.